Name: _____ Group _____

1)
a) A man invites his fiancée to a fine hotel for a Sunday brunch. They decide to meet in the lobby of the hotel between 11:30 am and 12 noon. If they arrive a random times during this period, what is the probability that they will meet within 10 minutes? (Hint: do this geometrically)

b) Suppose a random variables X and Y have a joint density given by:

$$f_{X,Y}(x,y) = \begin{cases} kxy & 0 < x < y < 2 \\ 0 & else \end{cases}$$

Find the constant k so that this function is a valid density.

c) Consider the following marginal densities for X and Y. Assume that X and Y are independent of each other of each.

$f_X(x) = \frac{1}{3}(x^2 - 2)$, $0 \le x < 3$, 0 else

$f_Y(y) = \frac{1}{12}(y + 3)$, $2 \le y < 4$, 0 else.

i) What is $f_{X,Y}(x,y)$?

ii) Determine the CDF.

2) Consider two electrical components, A and B, with respective lifetimes X and Y in days. Assume that a joint PDF of X and Y is

$$f_{X,Y}(x,y) = \begin{cases} 10e^{-(2x+5y)} & x > 0, y > 0 \\ 0 & else \end{cases}$$

a) Verify that this is a valid density.

b) Determine the CDF.

c) Determine the density from the CDF.

d) Find the marginal density of X, $f_X(x)$

e) Find the marginal density of Y, $f_Y(y)$

f) Are X and Y independent? Why or why not?

g) Calculate the conditional density of Y when X = x where x > 0.

h) What is the conditional probability that Y is between 0 and 3 given that x = 2.

3) A bank operates both a drive-up facility and a walk-up window. On a randomly selected day, let X = the proportion of time that the drive-up facility is in use (at least one customer is being served or waiting to be served) and Y = the proportion of time that the walk-up window is in use. The joint PDF is

$$f_{X,Y}(x,y) = \begin{cases} \frac{6}{5}(x+y^2) & 0 \le x \le 1, \quad 0 \le y \le 1 \\ 0 & else \end{cases}$$

a) Verify that this is a valid density.

b) Calculate P(2X < Y).

c) Determine the CDF.

e) What is $f_X(x)$?

f) What is $f_Y(y)$?

g) Are X and Y independent? Why or why not?

e) Calculate the conditional density of X when Y = y where $0 \le y \le 1$.

f) What is the conditional probability that X is between 0 and 0.4 given that y = 0.2.

4) Suppose X,Y have a joint density

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{9}(3-x)(2-y) & if\ 0 \le x \le 3\ and\ 0 \le y \le 2, \\ 0 & otherwise \end{cases}$$

a) Verify that this is a valid density.

b) Find P(1 ≤ X ≤ 2, 0 ≤ Y≤ 1).

c) Find P(X < Y).

d) Find P(X > 2Y).

f) Find the Joint CDF

g) Calculate the density from the CDF.

5) Determine the joint density when $F_{X,Y}(x,y) = x^2y/2 + xy^2/2$ when $0 < x < 1$ and $0 < y < 1$.

6) Consider random variables X and Y with joint density
$f_{X,Y}(x,y) = \frac{1}{8}(1-x^2)(3-y)\ \ for -1 \le x \le 1\ \ and -1 \le y \le 1$
and $f_{X,Y}(x,y) = 0$ otherwise.

a) Find the probability that X and Y are both negative.

b) Find the density of X.

c) Find the density of Y.

d) Are X and Y independent? (Answer this question using the definition 26.1, p.333).

e) Calculate the conditional density of X when Y = y where $-1 \le y \le 1$.

f) What is the conditional probability that X is between x < 0 given that y = -1.

g) What is P(X < 0)?

h) Why are the answers to f) and g) same?

7) Please state whether X and Y are independent in each of the following joint densities. No integrals are required in your explanations.

a) $f_{X,Y}(x,y) = \frac{12}{7}(xy + x^2)\ \ for\ 0 \le x \le 1, 0 \le y \le 1$ and $f_{X,Y}(x,y) = 0$ otherwise.

b) $f_{X,Y}(x,y) = \frac{3xy}{1250}\ \ for\ 0 \le x, 0\ \le y, x + y \le 10$ and $f_{X,Y}(x,y) = 0$ otherwise.

c) $f_{X,Y}(x,y) = \frac{9}{32}\sqrt{xy}\ \ for\ 0 \le x \le 1, 0\ \le y \le 4$ and $f_{X,Y}(x,y) = 0$ otherwise.

8) BONUS: A nut company markets cans of deluxe mixed nuts containing almonds, cashews and peanuts. Suppose the net weight of each can is exactly 1 lb, but the weight contribution of each type of nut is random. Because the three weights sum to 1, a joint probability model for any two gives all necessary information about the weight of the third type. Let X = the weight of almonds in a selected can and Y = the weight of cashews. The joint PDF is

$$f_{X,Y}(x,y) = \begin{cases} 24xy & 0 \le x \le 1, 0 \le y \le 1, x + y < 1 \\ 0 & else \end{cases}$$

a) Verify that this is a valid density.

b) What is the probability that there are greater than 0.25 lbs of peanuts in the can?

c) What is $f_X(x)$?

d) What is $f_Y(y)$?

e) Are X and Y independent? Why or why not?